11.3 Perimeter and Area of Similar Figures

Before You used ratios to find perimeters of similar figures.

Now You will use ratios to find areas of similar figures.

Why So you can apply similarity in cooking, as in Example 3.

Key Vocabulary
- **regular polygon,** *p. 43*
- **corresponding sides,** *p. 225*
- **similar polygons,** *p. 372*

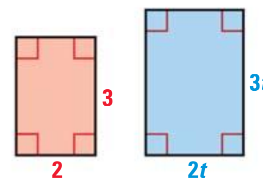
In Chapter 6 you learned that if two polygons are similar, then the ratio of their perimeters, or of any two corresponding lengths, is equal to the ratio of their corresponding side lengths. As shown below, the areas have a different ratio.

Ratio of perimeters

$$\frac{\text{Blue}}{\text{Red}} = \frac{10t}{10} = t$$

Ratio of areas

$$\frac{\text{Blue}}{\text{Red}} = \frac{6t^2}{6} = t^2$$



THEOREM — *For Your Notebook*

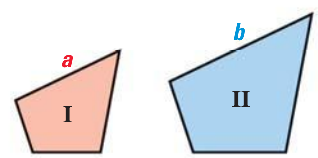
THEOREM 11.7 Areas of Similar Polygons

If two polygons are similar with the lengths of corresponding sides in the ratio of $a:b$, then the ratio of their areas is $a^2:b^2$.

$$\frac{\text{Side length of Polygon I}}{\text{Side length of Polygon II}} = \frac{a}{b}$$

$$\frac{\text{Area of Polygon I}}{\text{Area of Polygon II}} = \frac{a^2}{b^2}$$

Justification: Ex. 30, p. 742



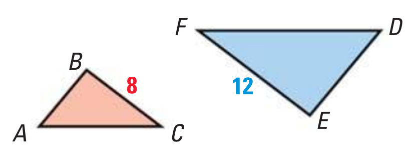
Polygon I ~ Polygon II

EXAMPLE 1 Find ratios of similar polygons

In the diagram, $\triangle ABC \sim \triangle DEF$. Find the indicated ratio.

a. Ratio (red to blue) of the perimeters

b. Ratio (red to blue) of the areas



Solution

The ratio of the lengths of corresponding sides is $\frac{8}{12} = \frac{2}{3}$, or $2:3$.

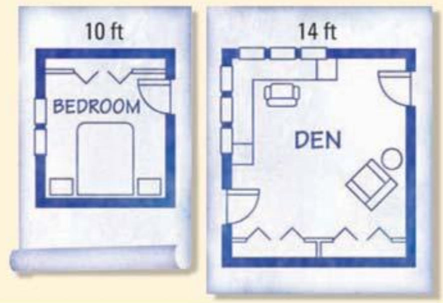
a. By Theorem 6.1 on page 374, the ratio of the perimeters is $2:3$.

b. By Theorem 11.7 above, the ratio of the areas is $2^2:3^2$, or $4:9$.

INTERPRET RATIOS
You can also compare the measures with fractions. The perimeter of $\triangle ABC$ is two thirds of the perimeter of $\triangle DEF$. The area of $\triangle ABC$ is four ninths of the area of $\triangle DEF$.

EXAMPLE 2 Standardized Test Practice

You are installing the same carpet in a bedroom and den. The floors of the rooms are similar. The carpet for the bedroom costs \$225. Carpet is sold by the square foot. How much does it cost to carpet the den?



Ⓐ \$115 Ⓑ \$161

Ⓒ \$315 Ⓓ \$441

USE ESTIMATION

The cost for the den is $\frac{49}{25}$ times the cost for the bedroom. Because $\frac{49}{25}$ is a little less than 2, the cost for the den is a little less than twice \$225. The only possible choice is D.

Solution

The ratio of a side length of the den to the corresponding side length of the bedroom is 14 : 10, or 7 : 5. So, the ratio of the areas is $7^2 : 5^2$, or 49 : 25. This ratio is also the ratio of the carpeting costs. Let x be the cost for the den.

$\frac{49}{25} = \frac{x}{225}$ ← **cost of carpet for den** / ← **cost of carpet for bedroom**

$x = 441$ **Solve for *x*.**

▶ It costs \$441 to carpet the den. The correct answer is D. Ⓐ Ⓑ Ⓒ Ⓓ

✓ GUIDED PRACTICE for Examples 1 and 2

1. The perimeter of $\triangle ABC$ is 16 feet, and its area is 64 feet. The perimeter of $\triangle DEF$ is 12 feet. Given $\triangle ABC \sim \triangle DEF$, find the ratio of the area of $\triangle ABC$ to the area of $\triangle DEF$. Then find the area of $\triangle DEF$.

EXAMPLE 3 Use a ratio of areas

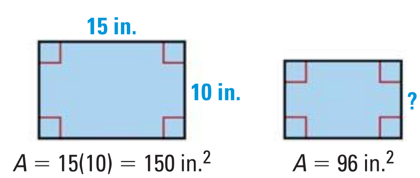
COOKING A large rectangular baking pan is 15 inches long and 10 inches wide. A smaller pan is similar to the large pan. The area of the smaller pan is 96 square inches. Find the width of the smaller pan.

ANOTHER WAY

For an alternative method for solving the problem in Example 3, turn to page 744 for the **Problem Solving Workshop**.

Solution

First draw a diagram to represent the problem. Label dimensions and areas.



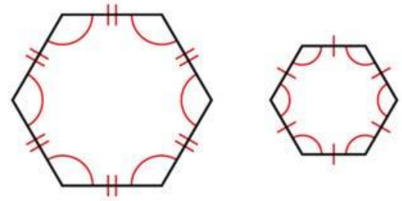
Then use Theorem 11.7. If the area ratio is $a^2 : b^2$, then the length ratio is $a : b$.

$\frac{\text{Area of smaller pan}}{\text{Area of large pan}} = \frac{96}{150} = \frac{16}{25}$ **Write ratio of known areas. Then simplify.**

$\frac{\text{Length in smaller pan}}{\text{Length in large pan}} = \frac{4}{5}$ **Find square root of area ratio.**

▶ Any length in the smaller pan is $\frac{4}{5}$, or 0.8, of the corresponding length in the large pan. So, the width of the smaller pan is 0.8(10 inches) = 8 inches.

REGULAR POLYGONS Consider two regular polygons with the same number of sides. All of the angles are congruent. The lengths of all pairs of corresponding sides are in the same ratio. So, any two such polygons are similar. Also, any two circles are similar.



EXAMPLE 4 Solve a multi-step problem

GAZEBO The floor of the gazebo shown is a regular octagon. Each side of the floor is 8 feet, and the area is about 309 square feet. You build a small model gazebo in the shape of a regular octagon. The perimeter of the floor of the model gazebo is 24 inches. Find the area of the floor of the model gazebo to the nearest tenth of a square inch.

Solution

All regular octagons are similar, so the floor of the model is similar to the floor of the full-sized gazebo.

ANOTHER WAY
In Step 1, instead of finding the perimeter of the full-sized and comparing perimeters, you can find the side length of the model and compare side lengths. $24 \div 8 = 3$, so the ratio of side lengths is $\frac{8 \text{ ft.}}{3 \text{ in.}} = \frac{96 \text{ in.}}{3 \text{ in.}} = \frac{32}{1}$.

STEP 1 **Find** the ratio of the lengths of the two floors by finding the ratio of the perimeters. Use the same units for both lengths in the ratio.

$$\frac{\text{Perimeter of full-sized}}{\text{Perimeter of model}} = \frac{8(8 \text{ ft})}{24 \text{ in.}} = \frac{64 \text{ ft}}{24 \text{ in.}} = \frac{64 \text{ ft}}{2 \text{ ft}} = \frac{32}{1}$$

So, the ratio of corresponding lengths (full-sized to model) is 32 : 1.

STEP 2 **Calculate** the area of the model gazebo's floor. Let x be this area.

$$\frac{(\text{Length in full-sized})^2}{(\text{Length in model})^2} = \frac{\text{Area of full-sized}}{\text{Area of model}}$$ **Theorem 11.7**

$$\frac{32^2}{1^2} = \frac{309 \text{ ft}^2}{x \text{ ft}^2}$$ **Substitute.**

$$1024x = 309$$ **Cross Products Property**

$$x \approx 0.302 \text{ ft}^2$$ **Solve for *x*.**

STEP 3 **Convert** the area to square inches.

$$0.302 \text{ ft}^2 \cdot \frac{144 \text{ in.}^2}{1 \text{ ft}^2} \approx 43.5 \text{ in.}^2$$

▶ The area of the floor of the model gazebo is about 43.5 square inches.

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✓ GUIDED PRACTICE for Examples 3 and 4

2. The ratio of the areas of two regular decagons is 20 : 36. What is the ratio of their corresponding side lengths in simplest radical form?

3. Rectangles I and II are similar. The perimeter of Rectangle I is 66 inches. Rectangle II is 35 feet long and 20 feet wide. Show the steps you would use to find the ratio of the areas and then find the area of Rectangle I.

11.3 EXERCISES

HOMEWORK KEY

◯ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 17, and 27

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 12, 18, 28, 32, and 33

SKILL PRACTICE

1. **VOCABULARY** Sketch two similar triangles. Use your sketch to explain what is meant by *corresponding side lengths.*

2. ★ **WRITING** Two regular *n*-gons are similar. The ratio of their side lengths is is 3 : 4. Do you need to know the value of *n* to find the ratio of the perimeters or the ratio of the areas of the polygons? *Explain.*

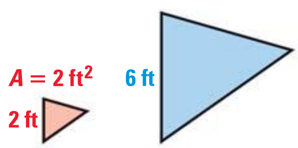
EXAMPLES 1 and 2 on pp. 737–738 for Exs. 3–8

FINDING RATIOS **Copy and complete the table of ratios for similar polygons.**

	Ratio of corresponding side lengths	Ratio of perimeters	Ratio of areas
3.	6 : 11	?	?
4.	?	20 : 36 = ?	?

RATIOS AND AREAS **Corresponding lengths in similar figures are given. Find the ratios (red to blue) of the perimeters and areas. Find the unknown area.**

5.



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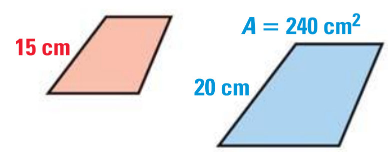

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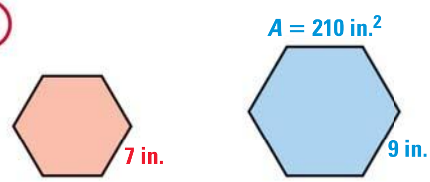

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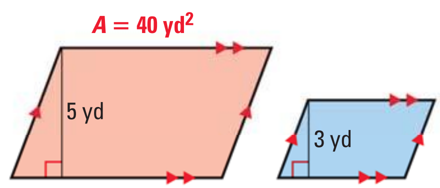


EXAMPLE 3 on p. 738 for Exs. 9–15

FINDING LENGTH RATIOS **The ratio of the areas of two similar figures is given. Write the ratio of the lengths of corresponding sides.**

9. Ratio of areas = 49 : 16

10. Ratio of areas = 16 : 121

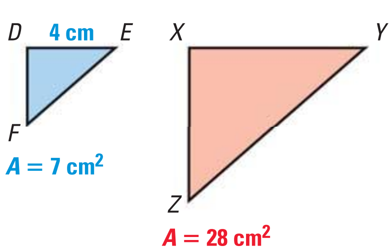
11. Ratio of areas = 121 : 144

12. ★ **MULTIPLE CHOICE** The area of $\triangle LMN$ is 18 ft^2 and the area of $\triangle FGH$ is 24 ft^2. If $\triangle LMN \sim \triangle FGH$, what is the ratio of *LM* to *FG*?

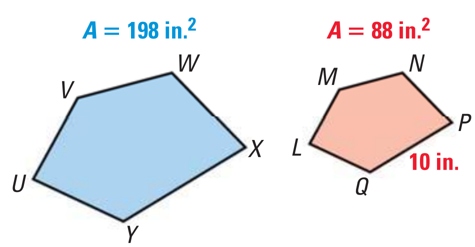
(A) 3 : 4 (B) 9 : 16 (C) $\sqrt{3} : 2$ (D) 4 : 3

FINDING SIDE LENGTHS **Use the given area to find *XY*.**

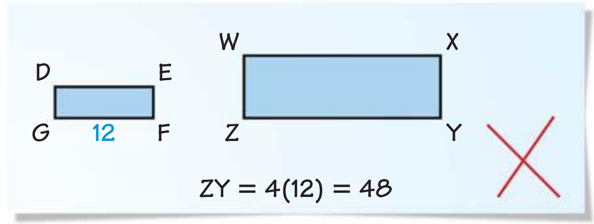
13. $\triangle DEF \sim \triangle XYZ$



14. $UVWXY \sim LMNPQ$



15. **ERROR ANALYSIS** In the diagram, Rectangles *DEFG* and *WXYZ* are similar. The ratio of the area of *DEFG* to the area of *WXYZ* is 1 : 4. *Describe* and correct the error in finding *ZY*.



EXAMPLE 4
on p. 739
for Exs. 16–17

16. **REGULAR PENTAGONS** Regular pentagon *QRSTU* has a side length of 12 centimeters and an area of about 248 square centimeters. Regular pentagon *VWXYZ* has a perimeter of 140 centimeters. Find its area.

17. **RHOMBUSES** Rhombuses *MNPQ* and *RSTU* are similar. The area of *RSTU* is 28 square feet. The diagonals of *MNPQ* are 25 feet long and 14 feet long. Find the area of *MNPQ*. Then use the ratio of the areas to find the lengths of the diagonals of *RSTU*.

18. ★ **SHORT RESPONSE** You enlarge the same figure three different ways. In each case, the enlarged figure is similar to the original. List the enlargements in order from smallest to largest. *Explain.*

Case 1 The side lengths of the original figure are multiplied by 3.
Case 2 The perimeter of the original figure is multiplied by 4.
Case 3 The area of the original figure is multiplied by 5.

REASONING In Exercises 19 and 20, copy and complete the statement using *always*, *sometimes*, or *never*. *Explain* your reasoning.

19. Doubling the side length of a square __?__ doubles the area.

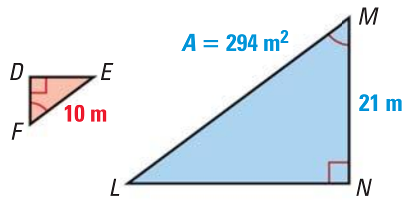
20. Two similar octagons __?__ have the same perimeter.

21. **FINDING AREA** The sides of $\triangle ABC$ are 4.5 feet, 7.5 feet, and 9 feet long. The area is about 17 square feet. *Explain* how to use the area of $\triangle ABC$ to find the area of a $\triangle DEF$ with side lengths 6 feet, 10 feet, and 12 feet.

22. **RECTANGLES** Rectangles *ABCD* and *DEFG* are similar. The length of *ABCD* is 24 feet and the perimeter is 84 square feet. The width of *DEFG* is 3 yards. Find the ratio of the area of *ABCD* to the area of *DEFG*.

SIMILAR TRIANGLES *Explain* why the red and blue triangles are similar. Find the ratio (red to blue) of the areas of the triangles. Show your steps.

23.



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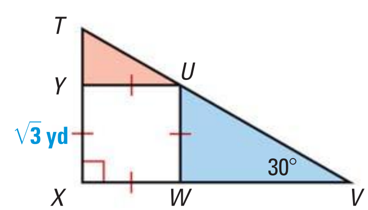
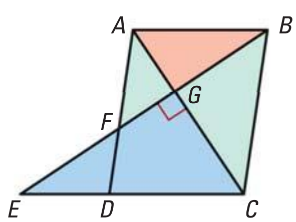


25. **CHALLENGE** In the diagram shown, *ABCD* is a parallelogram. The ratio of the area of $\triangle AGB$ to the area of $\triangle CGE$ is 9 : 25, $CG = 10$, and $GE = 15$.

a. Find *AG*, *GB*, *GF*, and *FE*. Show your methods.

b. Give two area ratios other than 9 : 25 or 25 : 9 for pairs of similar triangles in the figure. *Explain.*



PROBLEM SOLVING

26. **BANNER** Two rectangular banners from this year's music festival are shown. Organizers of next year's festival want to design a new banner that will be similar to the banner whose dimensions are given in the photograph. The length of the longest side of the new banner will be 5 feet. Find the area of the new banner.

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EXAMPLE 3 on p. 738 for Ex. 27

27. **PATIO** A new patio will be an irregular hexagon. The patio will have two long parallel sides and an area of 360 square feet. The area of a similar shaped patio is 250 square feet, and its long parallel sides are 12.5 feet apart. What will be the corresponding distance on the new patio?

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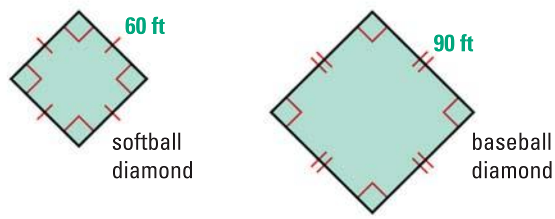
28. ★ **MULTIPLE CHOICE** You need 20 pounds of grass seed to plant grass inside the baseball diamond shown. About how many pounds do you need to plant grass inside the softball diamond?

Ⓐ 6 Ⓑ 9

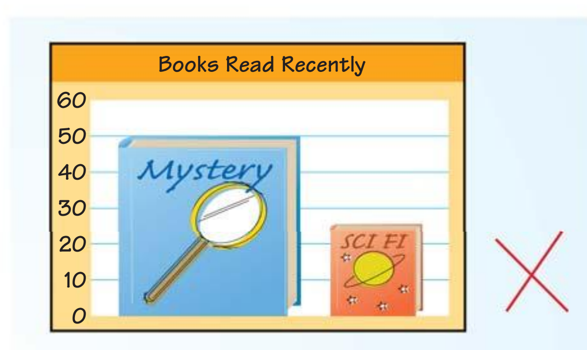
Ⓒ 13 Ⓓ 20



29. **MULTI-STEP PROBLEM** Use graph paper for parts (a) and (b).
    a. Draw a triangle and label its vertices. Find the area of the triangle.
    b. Mark and label the midpoints of each side of the triangle. Connect the midpoints to form a smaller triangle. Show that the larger and smaller triangles are similar. Then use the fact that the triangles are similar to find the area of the smaller triangle.

30. **JUSTIFYING THEOREM 11.7** Choose a type of polygon for which you know the area formula. Use algebra and the area formula to prove Theorem 11.7 for that polygon. (*Hint:* Use the ratio for the corresponding side lengths in two similar polygons to express each dimension in one polygon as $\frac{a}{b}$ times the corresponding dimension in the other polygon.)

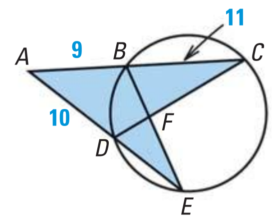
31. **MISLEADING GRAPHS** A student wants to show that the students in a science class prefer mysteries to science fiction books. Over a two month period, the students in the class read 50 mysteries, but only 25 science fiction books. The student makes a bar graph of these data. *Explain* why the graph is visually misleading. Show how the student could redraw the bar graph.



○ = WORKED-OUT SOLUTIONS on p. WS1 ★ = STANDARDIZED TEST PRACTICE

32. ★ **OPEN-ENDED MATH** The ratio of the areas of two similar polygons is 9 : 6. Draw two polygons that fit this description. Find the ratio of their perimeters. Then write the ratio in simplest radical form.

33. ★ **EXTENDED RESPONSE** Use the diagram shown at the right.

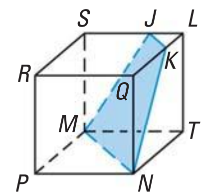


a. Name as many pairs of similar triangles as you can. *Explain* your reasoning.

b. Find the ratio of the areas for one pair of similar triangles.

c. Show two ways to find the length of $\overline{DE}$.

34. **CHALLENGE** In the diagram, the solid figure is a cube. Quadrilateral *JKNM* is on a plane that cuts through the cube, with $JL = KL$.



a. *Explain* how you know that $\triangle JKL \sim \triangle MNP$.

b. Suppose $\frac{JK}{MN} = \frac{1}{3}$. Find the ratio of the area of $\triangle JKL$ to the area of one face of the cube.

c. Find the ratio of the area of $\triangle JKL$ to the area of pentagon *JKQRS*.

MIXED REVIEW

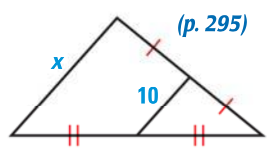
PREVIEW
Prepare for Lesson 11.4 in Exs. 35–38.

Find the circumference of the circle with the given radius *r* or diameter *d*. Use $\pi \approx 3.14$. Round your answers to the nearest hundredth. *(p. 49)*

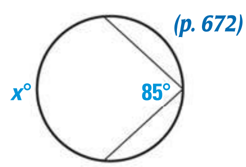
35. $d = 4$ cm   36. $d = 10$ ft   37. $r = 2.5$ yd   38. $r = 3.1$ m

Find the value of *x*.

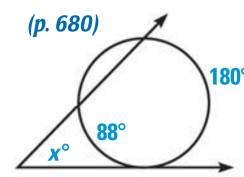
39. *(p. 295)*



40. *(p. 672)*



41. *(p. 680)*

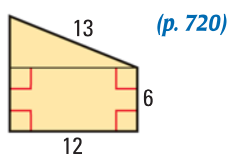


QUIZ *for Lessons 11.1–11.3*

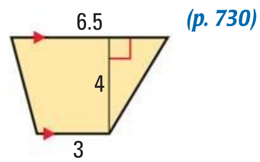
1. The height of ▱*ABCD* is 3 times its base. Its area is 108 square feet. Find the base and the height. *(p. 720)*

Find the area of the figure.

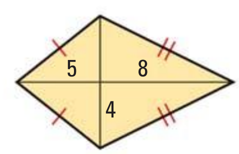
2. *(p. 720)*



3. *(p. 730)*



4. *(p. 730)*



5. The ratio of the lengths of corresponding sides of two similar heptagons is 7 : 20. Find the ratio of their perimeters and their areas. *(p. 737)*

6. Triangles *PQR* and *XYZ* are similar. The area of $\triangle PQR$ is 1200 ft^2 and the area of $\triangle XYZ$ is 48 ft^2. Given $PQ = 50$ ft, find *XY*. *(p. 737)*